FINITE ELEMENT ANALYSIS OF INORGANIC SLUDGE CONSOLIDATION IN A WASTE DISPOSAL FACILITY WITH DRAINAGE

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ABSTRACT
Simplified finite element analysis is performed for the two-dimensional consolidation of a sludge disposed in a pond with lateral and bottom draining boundaries. The effect of the coefficient of consolidation and of the drainage capacity of the boundaries on the settlement rate are investigated for a case example. Hyperbolic regression is used to analyze the results.

RÉSUMÉ
On présente une analyse simplifiée, par la méthode des éléments finis, de la consolidation bidimensionnelle d'un dépôt de déchets boueux dont les limites latérales et le fond sont drainants. L'influence du coefficient de consolidation de la boue et de la capacité des drainages est étudiée au moyen d'un exemple de cas. Les résultats sont analysés par la méthode de régression hyperbolique.

1. INTRODUCTION

Large amounts of compressible saturated waste are disposed under the form of sludge or slurry in ponds or in excavated landfills. Because of the large volumes involved, assessing the settlement upon time of such refuses has significant environmental and economical bearings. Most often in current practice, waste settlement analysis rely on empirical formulas extrapolated from previous monitoring data rather than established rational theories. In absence of chemical reaction and biodegradation, theoretical assessment of sludge settlement would consider a sequence of three mechanisms that are sedimentation, hydrodynamic consolidation under self-weight, and secondary (creep) compression. The popularity of empirical approaches is supported by a number of factors including, convenience of using simple formulas fitted to field data, difficulty to determine the constitutive behaviour of the material, and limitation of theories. For instance, in Terzaghi's classical consolidation theory (1925), one dimensional vertical drainage and uniform compression are assumed. This is a considerable limitation to modeling of field situations which are, in general, three dimensional with respect to drainage and deformation. This discrepancy creates also a bias in the determination of the parameter governing the time-rate of consolidation, Cv. The coefficient of consolidation, Cv is determined using one dimensional laboratory tests, then it is input in computational models of field consolidation resulting, not surprisingly, in underestimation of the time-rate of settlement. In concept, Biot's three-dimensional theory (1941) addresses this issue, but it is not practical enough for irregular boundary problems with material non-linearity. Other limitations of Terzaghi's theory including, linearity of the constitutive parameters, restriction to small strain and thin homogeneous layers, have been recognized and addressed in subsequent studies (e.g. Schiffman and Gibson, 1964, Schiffman et al, 1969, Gibson et al, 1981, and Duncan, 1993). More general theoretical frameworks have been also elaborated to account for the sedimentation stage (e.g. Koppula and Morgenstern, 1982, Yong et al, 1983) and secondary compression (e.g. Gibson and Lo, 1961, Bourdeau, 2001). The particular problem of self weight settlement has been analysed by Been and Sills (1981), Schwieger and Schuppener (1995), and Toorman (1999). However, these various improvements upon the original theory are of practical use only for one-dimensional analysis. As part of these developments, a promising semi-empirical procedure to predict consolidation settlement of very soft and non-homogeneous clay deposits has been proposed, the hyperbolic method (Tan et al, 1991). According to this method, the settlement (s) versus time (t) relationship approaches a straight line in a (t/s) versus (t) diagram. The method has been shown to be in agreement with computation results of infinitesimal and finite strain one-dimensional consolidation, as well as with field observation in a settling pond (Tan et al, 1991). In the present paper, the hyperbolic method is tested for its consistency with time-rate of settlement computed in a two-dimensional model of sludge consolidation.

2. MODEL FORMULATION

Chemical or biological factors are not considered herein. The geometry of the example problem being analysed is shown in Figure.1. Hydrodynamic consolidation of a saturated sludge disposed in a trapezoidal cross-section pond or landfill is modeled as a two-dimensional plane strain problem. In principle, the analysis consists in solving two coupled boundary-initial value problems, the transient flow of excess pore water and the self weight-induced deformation. In the present simplified formulation, the fluid flow is modeled using a two-dimensional extension of Terzaghi's consolidation equation,

$$\partial^2P/\partial x^2 + \partial^2P/\partial y^2 = (1/Cv)\partial P/\partial t$$

where P (x,y,t) is excess pore pressure, x and y are Eulerian coordinates and t is the time; Cv is the coefficient of consolidation which may be a function of the stress level. Deformation of the sludge mass results from increase of effective stress consecutive to dissipation of excess pore pressure. The principle of effective stress provides the

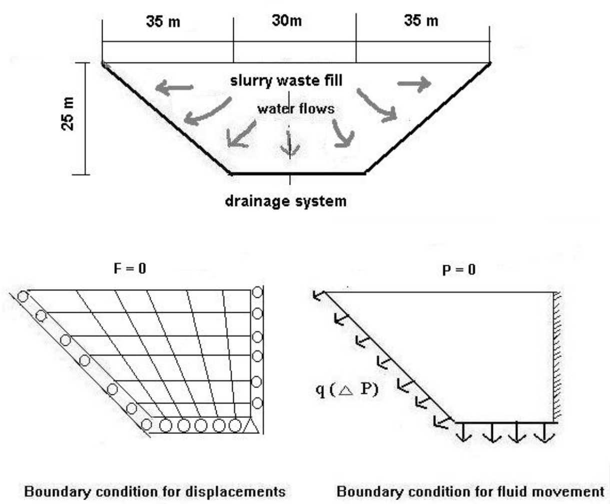

Figure 1. Model configuration and boundary conditions for the FE analysis (F : gravitational force ; P : excess pore pressure ; q (ΔP) : flow rate at convective boundary)

coupling relationship between flow and deformation equations.

The boundary conditions shown schematically in Figure 1 take advantage of the problem symmetry. The sludge is allowed to slide without resistance along the boundaries. The hydraulic boundary conditions are more complex; while the upper surface boundary is kept at zero excess pore pressure, the bottom and lateral boundaries need to be represented by saturated drainage layers with a cross-flux proportional to the hydraulic gradient, according to Darcy's law.

Initial conditions are represented by the distribution of excess pore pressure in the sludge at the beginning of the consolidation process. It is assumed that, after the sludge pond is filled and sedimentation has taken place, consolidation will start while the effective stress is still zero. This means the initial excess pore pressure, being equal to the difference between final and initial effective stresses, can be computed as the stress resulting from the buoyant self weight of the sludge. In the present case, the average elastic stress is used to determine the initial excess pore pressure at a point.

3. SOLUTION METHOD

The finite element method is used to solve the consolidation equations and to compute the resulting deformation field. The software, ANSYS, utilized in this study, has capability to solve heat conduction problems. Since both heat conduction and consolidation are modeled by the transient form of a diffusion equation, ANSYS can also be used for solving the consolidation problem, provided the appropriate coefficients are introduced.

The drainage boundary condition is written

$$q = K_D/L\ (P_b - P_0)$$

where q is the flow rate across the boundary, K_D is the drainage layer hydraulic conductivity; L is the drainage layer thickness ; P_b , P_o denote the excess pore pressure at inner and outer surfaces of the layer respectively. In the context of the solution using ANSYS, this is the analog of a heat convection boundary.

Using the finite element method, excess pore pressures are computed at selected time steps. The evolution upon time of the consolidation process and of the corresponding relative settlement is quantified by the average degree of consolidation,

$$Sr\ (t) = 1 - [\iint P\ (x,y,t)\ dx\ dy\ /\ \iint P(x,y,0)\ dx\ dy\ ]$$

where P (x,y,t) is the excess pore pressure to be integrated over the whole cross section at time t . Sr varies between zero, at the start of consolidation, and one at the end.

4. MATERIAL PARAMETERS

The coefficient of consolidation is a linear function of the hydraulic conductivity, k, and is inversely proportional to the sludge compressibility represented by its coefficient of volume change, m_v,

$$Cv = k/(\gamma_w m_v)$$

where γ_w is the unit weight of the pore fluid.

Because both k and m_v decrease during the course of consolidation (as a result of increasing state of compaction), Cv is generally assumed constant for practical purposes. However, there is experimental evidence that Cv can vary in function of the effective confining pressure (e.g. Duncan, 1993, Robinson and Allan, 1996). In the present study, both possibility of Cv increasing or decreasing in function of the effective stress are considered. Linear variations of Cv in function of the average effective stress are simulated by updating the parameter at computational time steps. As the effective stress ranges between 20 and 80 kPa, two possible ranges of variation "low values" between $9*10^{-3}$ m^2/day and $4.32*10^{-2}$ m^2/day ($5*10^{-7}$ m^2/sec), and "high values" between 0.09 m^2/day and 0.432 m^2/day ($5*10^{-6}$ m^2/sec) are considered for Cv. Within each of these two ranges, Cv is assumed either increasing or decreasing in function of effective stress. These data account for the fact that field values of Cv are typically larger than laboratory test results. Corresponding data used for other sludge material parameters are, 1.00 Mpa^{-1} for the coefficient of volume change and 9 kN/m^3 for the buoyant unit weight. These data are consistent with typical properties of industrial refuse (e.g. Sharma ad Lewis, 1995).

In order to investigate the effect of the drainage boundaries on the consolidation, a sensitivity study is performed on the hydraulic conductivity, K_d, controlling the convective flux across the drainage layer. The data used for K_d range between $1*10^{-7}$ m/sec and $1*10^{-5}$ m/sec.

5. RESULTS AND DISCUSSION OF CASE EXAMPLE

The final effective stress and displacement fields computed when consolidation is completed are shown in Figure 2. where part (A) represents the distribution of the major principal effective stress and part (B) the displacement patterns. The effects of the two-dimensional geometry and of the sliding lateral boundaries are particularly evident in the displacement field but are also apparent in the stress release in the upper left area and concentration at the toe of the slope. It is noted that the surface of the refuse remains horizontal after deformation.

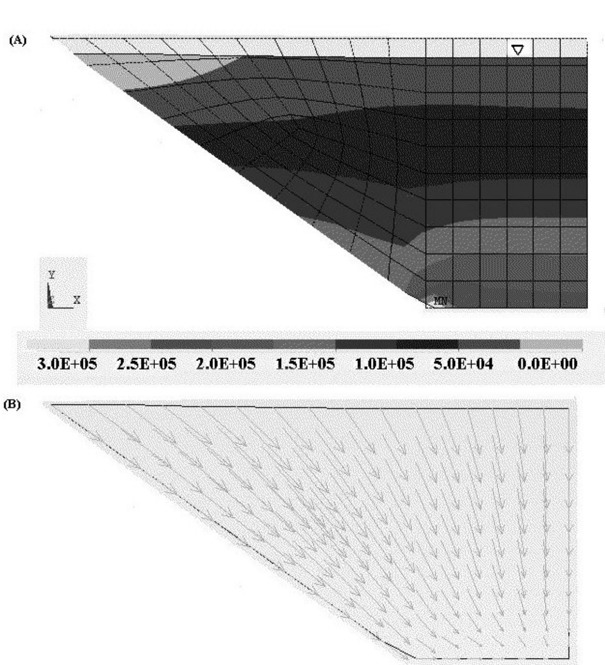

Figure 2 (A) Major principal effective stress distribution (in Pa) and (B) the deformation at the end of consolidation (maximum principal stress : $3.21*10^5$ Pa at left bottom corner ; maximum displacement : 2.12 m at left top corner)

The evolution of excess pore pressure is presented in Figure 3 for three selected stages, first the initial situation, then after 20 days and 100 days. The initial distribution is also influenced by the geometry and exhibits a peak at the toe of the slope. The effect of the lateral drainage is significant on the pore pressure dissipation, once consolidation occurs. After a larger time has elapsed, a strong bulb of excess pore pressure remains at the core of the sludge mass. The results shown in Figure 3 were obtained with Cv in the “high values” range. Little difference was obtained when Cv is decreasing in function of the stress instead of being kept constant. On the contrary, increasing values of Cv with the stress was found to result in delayed dissipation of excess pore pressure. In Figure 4 and Figure 5 are summarized the results of a sensitivity study on the drainage capacity of the boundaries. This also serves to assess the validity of the hyperbolic method in two-dimensional consolidation problems. A first set of computations was performed with Cv in the “ high value” range (0.09 to 0.432 m^2/day) and K_d between $1*10^{-7}$ and $1*10^{-5}$ m/sec. Figure 4(A) is a plot of the average degree of consolidation, or the corresponding relative amount of settlement in function of time. There is practically no difference between the curves obtained with Cv = const = 0.432 m^2/day and those computed with Cv decreasing in function of stress. On the contrary, computations performed using Cv increasing within the same range in function of stress result in slower consolidation and settlement rates. The influence of K_d is significant, especially in the increasing Cv situation. For the data being used, full consolidation is almost achieved after 1000 days except in one case. The second set of computation results, performed with Cv in the

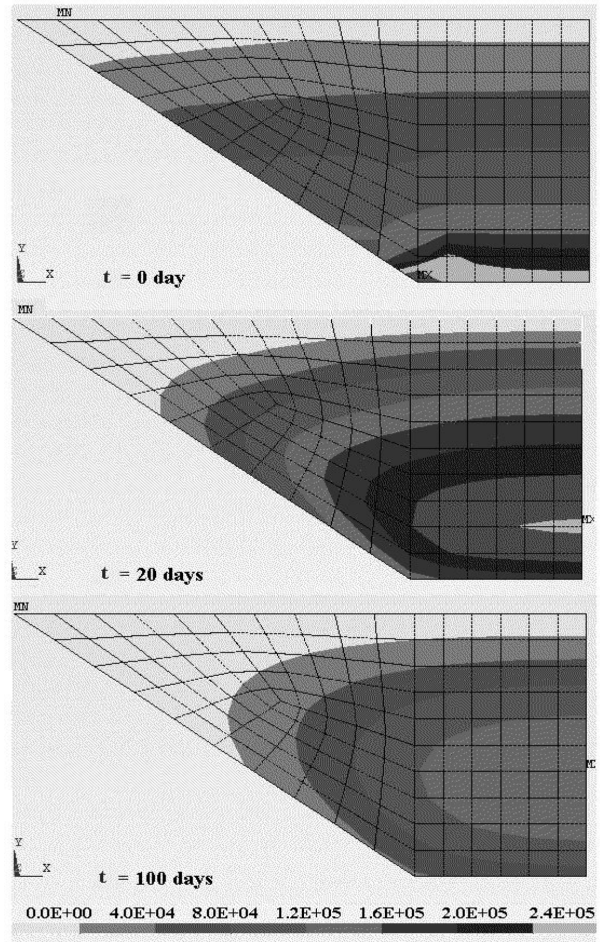

Figure 3 Excess pore pressure dissipation during consolidation (unit : Pa) (Cv,max = 0.432 m^2/day ;K_d = 10^{-6} m/sec) Top, middle and bottom figures show the distributions after 0 day, 20 days and100 days, respectively.

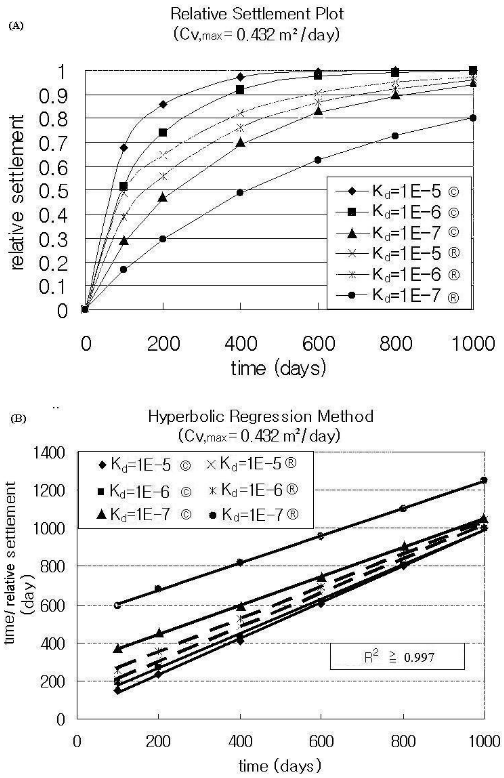

Figure 4 (A) Relative settlement in function of time and (B) hyperbolic regression, computed using “high value” range of the coefficient of consolidation; K_d : drainage hydraulic conductivity (m/sec); © indicates Cv constant (=Cv,max) or decreasing with effective stress increased, ® indicates Cv increasing with effective stress increased

“low value” range ($9*10^{-3}$ to $4.32*10^{-2}$ m^2/day) is presented in Figure 5. The plots in Figure 5(A) show a pattern similar to Figure 4(A) but with a much slower consolidation. Only 40% to 70 % of the final settlement is achieved after 1000 days. Sensitivity to the boundary drainage capacity is also lesser than in the previous case. This suggest that the permeability of the sludge would be the controlling factor in this case and increasing the boundary drainage capacity would not be most effective way to accelerate consolidation.

According to Tan et al (1991) the relationship between the inverse of the settlement rate and time is approximately linear, and can be expressed as

$$t/S = 1/\rho_0 + t/S_{ult}$$

where S is the settlement, t the time, ρ_0 the initial settlement rate (at t=0), and S_{ult} is the final settlement at the end of consolidation. If the proposed relationship is a good approximation for settlements observed over a finite period of time (or computed), then a regression analysis can

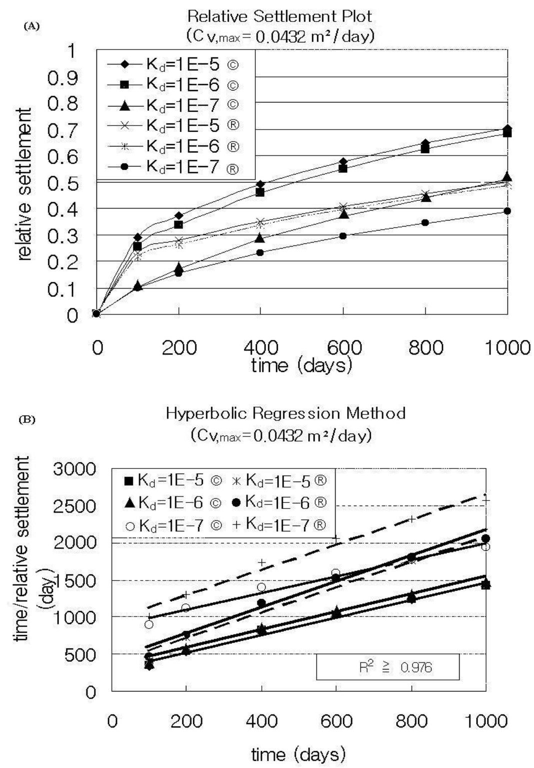

Figure 5 (A) Relative settlement in function of time and (B) its hyperbolic regression for the computation using “low value” range of the coefficient of consolidation (notations are the same to Figure 4)

provide an estimation of the ultimate settlement. Since the relative settlement is $Sr=S/S_{ult}$, the above equation can be rewritten

$$t/Sr = S_{ult}/\rho_0 + \alpha t$$

where the slope α is equal to one if the method is correct.

Table 1 Regression results of slope (α) using the hyperbolic method

K_d (m/sec)	Cv=0.432 m^2/day	Cv=0.0432 m^2/day
1.E-7	**0.757**[1] (0.723)[2]	**1.117** (1.707)
1.E-6	**0.898** (0.863)	**1.174** (1.736)
1.E-5	**0.949** (0.901)	**1.175** (1.719)

[1]Bold numbers are for Cv = constant or Cv decreasing with effective stress and [2]numbers in parentheses are for Cv increasing with effective stress

The plots in Figure 4(B) and 5(B) show the shape of the function is a good fit to the data. In Table 1 are summarized the regression analysis results.

For cases computed with constant or decreasing Cv, all the hyperbolic regressions indicate a slope coefficient α within a relatively narrow range around the target value. However, when Cv is increasing with stress the values of α differ significantly from one for the second series of computation where Cv is in the " low value" range.

6. CONCLUSIONS

From the analysis performed using the finite element method for a case example of sludge disposal settlement, the following observations are made:

a) The drainage provided by lateral boundaries has significant effect on the consolidation rate. The drainage capacity of the boundary layers is an important factor. Thus, Potential clogging of drainage material would significantly reduce the consolidation rate.
b) Dependency of the coefficient of consolidation on the effective stress is an important factor if Cv increases with stress. If it decreases while the stress increases, then computing with Cv=const=$Cv_{,max}$ is a sufficient approximation.
c) The hyperbolic method proposed by Tan et al (1991), for reducing settlement vs. time data and extrapolating the final settlement, seems applicable also in the case of two-dimensional consolidation in most of the cases computed, but discrepancies are found in some of the examples with Cv increasing with effective stress. Hence, the method should be used with caution.

It is noted that the analysis presented herein was based on Terzaghi's classical theory of small strain consolidation for thin layers. Further investigation is needed to verify that the above observations are still valid when a more accurate analysis is performed using a finite strain consolidation model.

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